LINKÖPINGS TEKNISKA HÖGSKOLA
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Tentamen TATA 27/TEN1 Partial Differential Equations
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You can use on this examination tables of formulas in TATA27. No calculators.

1. Solve the following initial boundary value problem for the wave equation on the half-line:
$$u_{tt} = 4u_{xx} \quad \text{for } t > 0 \text{ and } x > 0$$
with the boundary condition
$$u(0,t) = 1 \quad \text{for } t > 0$$
and with the initial conditions
$$u(x,0) = x \quad \text{and} \quad u_t(x,0) = x^2 \quad \text{for } x > 0.$$
Calculate $u(1,1)$.

2. Solve the following initial-boundary value problem for the heat equation:
$$u_t = 4u_{xx} \quad \text{for } 0 < x < 2 \text{ and } t > 0,$$
$$u(0,t) = 1 \text{ and } u_x(2,t) = -1 \text{ for } t > 0$$
and
$$u(x,0) = 0 \text{ for } 0 < x < 2.$$

3. Consider the function $u = u(x,t)$ which satisfies the Poisson equation
$$u_{xx} + u_{yy} = 4 \text{ for } 0 < x < 1 \text{ and } 0 < y < 2$$
and is subject to the Dirichlet boundary conditions
$$u(x,0) = x \text{ for } 0 < x < 1, \ u(1,y) = 2y + 1 \quad \text{for } 0 < y < 2,$$
$$u(x,2) = 3x + 2 \text{ for } 0 < x < 1 \text{ and } u(0,y) = y \quad \text{for } 0 < y < 2.$$
Show that
$$-2 + x^2 + y^2 \le u(x,y) \le 1 + x^2 + y^2.$$

4. Find the function $u(x)$ that makes the integral
$$\int_1^4 (u'^2 + u^2 + e^x u)dx$$
minimal subject to the constrains $u(1) = 0$ and $u(4) = 1$.

5. Calculate the second derivative of $f(x) = |x^2 - 5x + 6|$.

6. Show that the problem
$$u_{tt} - u_{xx} - u_{yy} = 1 \quad \text{for } x^2 + y^2 < 2 \text{ and } t > 0,$$
$$u(x,y,t) = 0 \quad \text{for } x^2 + y^2 = 2, \ t > 0$$
and
$$u(x,0) = u_t(x,0) = 0 \quad \text{for } x^2 + y^2 < 2$$
has at most one solution.