LINKÖPINGS TEKNISKA HÖGSKOLA
Matematiska institutionen
Vladimir Kozlov

**Tentamen TATA 27 Partial Differential Equations**
**14 January 2010, 14-19**

You can use on this examination Formelsamlingen i partiella diff.ekv. för M3 av L-E Andersson. No calculators.

1. Solve the following Dirichlet problem for the heat equation on the half-line:
$$u_t = 2u_{xx} \quad \text{for } t > 0 \text{ and } x > 0$$
with the boundary condition
$$u(0,t) = 1 \quad \text{for } t > 0$$
together with the initial condition
$$u(x,0) = x^2 \quad \text{for } x > 0.$$
Show that $u_x(0,1) > 0$.

2. Solve the following boundary value problem for the wave equation:
$$u_{tt} - 3u_{xx} = 0 \quad \text{for } 0 < t \text{ and } 0 < x < 3,$$
$$u(0,t) = 1 \text{ and } u(3,t) = 0 \text{ for } 0 < t$$
and
$$u(x,0) = u_t(x,0) = 0 \quad \text{for } 0 < x < 3.$$

3. Consider the function $u = u(x,t)$ which satisfies the Poisson equation
$$u_{xx} + u_{yy} = 1 \text{ for } 0 < x < 4 \text{ and } 0 < y < 2$$
the Dirichlet boundary conditions
$$u(0,y) = u(4,y) = 0 \quad \text{for } 0 < y < 2$$
and
$$u(x,0) = 1 \quad \text{and} \quad u(x,2) = 2$$
Show that
$$\frac{x^2}{2} - 2x \le u(x,y) \le 4 + \frac{x^2}{2} - 2x \quad \text{for } 0 < x < 4 \text{ and } 0 < y < 2.$$

4. Find the function $y = u(x)$ that makes the integral

$$\int_0^2 (u'^2 + 2xu + u^2)dx$$

minimal subject to the constrains $u(0) = 0$ and $u'(2) = 1$.

5. Calculate the second derivative of $f(x) = |x^2 + x - 2|$ in distributional sense.

6. Show that the problem

$$u_{xx} + u_{yy} = 2 \quad \text{for } 1 < x < 2 \text{ and } 1 < y < 2,$$

$$u(1, y) = u_x(2, y) = 1 \quad \text{for } 1 < y < 2,$$
$$u(x, 1) = u(x, 2) = 0 \quad \text{for } 1 < x < 2$$

has at most one solution.