LINKÖPINGS TEKNISKA HÖGSKOLA
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Tentamen TATA 27 Partial Differential Equations
4 June 2009, 8-13

You can use on this examination Formelsamlingen i partiella diff.ekv. för M3 av L-E Andersson. No calculators.

1. Solve the following Dirichlet problem for the wave equation on the half-line:
$$u_{tt} = 2u_{xx} \quad \text{for } t > 0 \text{ and } x > 0$$
and
$$u(0,t) = 1 \quad \text{for } t > 0,$$
supplied with the initial conditions
$$u(x,0) = e^x \;\text{ and }\; u_t(x,0) = 1 \;\text{ for } x > 0.$$
Calculate $u(1,\sqrt{2})$.

2. Solve the following boundary value problem for the Laplace equation:
$$u_{xx} + u_{yy} = 0 \quad \text{for } 0 < x < 1 \text{ and } 0 < y < 3,$$
$$u_y(x,0) = u(x,3) = 0 \quad \text{for } 0 < x < 1$$
and
$$u(0,y) = 1 \;\text{ and }\; u(1,y) = 0 \;\text{ for } 0 < y < 3.$$

3. Consider the function $u = u(x,t)$ which satisfies the heat equation
$$u_t - u_{xx} = 1 \;\; 0 < x < 4 \text{ and } t > 0$$
the Dirichlet boundary conditions
$$u(0,t) = u(4,t) = 0 \quad \text{for } t > 0,$$
and the initial condition $u(x,0) = 0$ for $0 < x < 4$. Show that
$$-2 \le u(x,y) \le 2 \quad \text{for } 0 \le x \le 4 \text{ and } t \ge 0.$$

4. Find the function $y = u(x)$ that makes the integral
$$\int_0^2 (u'^2 + xu + 2u^2)dx$$
minimal subject to the constrains $u(0) = 0$ and $u'(2) = 0$.

5. Calculate the second derivative in distributional sense of the function $f(x)$, which is defined as follows: $f(x) = 0$ for $x \leq -\pi$, $f(x) = \sin x$ for $-\pi < x \leq \pi/2$ and $f(x) = e^x$ for $x > \pi/2$.

6. Show that the problem

$$u_{xx} + u_{yy} = 1 \quad \text{for } 1 < x < 2 \text{ and } 3 < y < 6,$$

$$u(1, y) = u_x(2, y) = 0 \quad \text{for } 3 < y < 6,$$
$$u(x, 3) = u(x, 6) = 1 \quad \text{for } 1 < x < 2$$

has at most one solution.