LINKÖPINGS TEKNISKA HÖGSKOLA
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Tentamen TATA 27 Partial Differential Equations
15 January 2009, 14-19

You can use on this examination Formelsamlingen i partiella diff.ekv. för M3 av L-E Andersson. No calculators.

1. Consider the following Neumann problem for the wave equation on the half-line:
$$u_{tt} = 4u_{xx} \quad \text{for } t > 0 \text{ and } x > 0$$
and
$$u_x(0,t) = 0 \quad \text{for } t > 0,$$
supplied with the initial condition
$$u(x,0) = u_t(x,0) = \begin{cases} 1 - |x-1| & \text{for } 0 < x < 2 \\ 0 & \text{for } x \geq 2 \end{cases}$$
Find the solution to this problem and calculate $u(1,1)$.

2. Solve the following boundary value problem for the heat equation:
$$u_t = 2u_{xx} \quad \text{for } 0 < x < 2 \text{ and } t > 0,$$
$$u(0,t) = 0,\ u_x(2,t) = 1 \quad \text{for } t > 0$$
and
$$u(x,0) = 1 \quad \text{for } 0 < x < 2.$$

3. Consider the function $u = u(x,t)$ which satisfies the Poisson equation
$$u_{xx} + u_{yy} = 1$$
in the disk $x^2 + y^2 < 4$ and the Dirichlet boundary condition:
$$u(x,y) = xy \quad \text{for } x^2 + y^2 = 4.$$
Show that
$$\frac{x^2+y^2}{4} - 3 \leq u(x,y) \leq \frac{x^2+y^2}{4} + 1 \quad \text{for } x^2 + y^2 \leq 4.$$

4. Find the curve $y = u(x)$ that makes the integral
$$\int_0^1 (u'^2 + xu)dx$$
minimal subject to the constrains $u(0) = 0$ and $u(1) = 1$.

5. Calculate the second derivative of $f(x) = |\sin x|$ in distributional sense.

6. Show that the problem

$$u_t - u_{xx} = 0 \quad \text{for } 1 < x < 2 \text{ and } t > 0,$$

$$u(1,t) = u_x(2,t) = 0 \quad \text{for } t > 0,$$

$$u(x,0) = 1 \quad \text{for } 1 < x < 2$$

has at most one solution.