LINKÖPINGS TEKNISKA HÖGSKOLA
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Svar till Tentamen TATA 27 Partial Differential Equations
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1.
$$u(x,t) = 1 + \frac{1}{2}\Big(e^{x+\sqrt{2}t} + e^{x-\sqrt{2}t} - 2\Big) + t$$

for $x - \sqrt{2}t > 0$ and

$$u(x,t) = 1 + \frac{1}{2}\Big(e^{x+\sqrt{2}t} - e^{\sqrt{2}t-x}\Big) + \frac{x}{\sqrt{2}}$$

for $x - \sqrt{2}t < 0$. Therefore, $u(1,\sqrt{2}) = 1 + (e^3 - e)/2 + 1/\sqrt{2}$.

2.
$$u(x,y) = \sum_{k=0}^{\infty} a_k (e^{\lambda_k(x-1)} - e^{-\lambda_k(x-1)}) \cos \lambda_k y,$$

where

$$\lambda_k = \frac{\pi(2k+1)}{6}$$

and

$$a_k = \frac{2\sin\lambda_k}{\lambda_k(e^{-\lambda_k} - e^{\lambda_k})}$$

3. Apply the maximum principle to the function

$$u(x,t) + (x-4)x/2$$

4.
$$u(x) = \frac{e^{\sqrt{2}x} - e^{-\sqrt{2}x}}{4\sqrt{2}(e^{2\sqrt{2}} + e^{-2\sqrt{2}})} - \frac{1}{4}x.$$

5.
$$f''(x) = h(x) - \delta(x+\pi) + (e^{\pi/2} - 1)\delta'(x - \pi/2) + e^{\pi/2}\delta(x - \pi/2),$$

where $h(x) = 0$ for $x \le -\pi$, $h(x) = -\sin x$ for $-\pi < x \le \pi/2$ and $h(x) = e^x$ for $x > \pi/2$.

6. If u_2 and u_1 are two solutions to the problem then use the first Green identity for the difference $u = u_2 - u_1$.