LINKÖPINGS TEKNISKA HÖGSKOLA
Matematiska institutionen
Vladimir Kozlov

Svar till Tentamen TATA 27 Partial Differential Equations
14 January 2008, 14-19

1.
$$u(1/2,1/3)=\frac{2-\sqrt{3}}{12}$$

2.
$$u(x,y)=\sum_{k=0}^{\infty}a_k\Big(e^{\sqrt{\lambda_k}(x-3)}-e^{-\sqrt{\lambda_k}(x-3)}\Big)\cos\lambda_k y,$$

where
$$\lambda_k=\frac{\pi}{2}\Big(k+\frac{1}{2}\Big)$$

and
$$a_k\Big(e^{-\sqrt{\lambda_k}3}-e^{\sqrt{\lambda_k}3}\Big)=\int_0^2(y^2-4)\cos\lambda_k y dy=-\frac{2}{\lambda_k^3}\sin 2\lambda_k$$

3. Using the maximum principle for the heat equation and for the function $v(x,t)=u(x,t)-x(1-x)/2$, we obtain the inequalities $-1/8\le v(x,t)\le 0$, which imply
$$-1/8+x(1-x)/2\le u(x,t)\le x(1-x)/2.$$
Using the maximum principle for the function $w(x,t)=u(x,t)-t$ we obtain that $-t\le u(x,\tau)\le 0$ for $0\le\tau\le t$, which implies that u is nonnegative.

4. The Euler equation is
$$2u=6u''+1$$
and its general solution is
$$u=\frac{1}{2}+ae^{x/\sqrt{3}}+be^{-x/\sqrt{3}}.$$
Using the boundary conditions for u we obtain
$$b=\Big(\frac{3}{2}-\frac{1}{2}e^{1/\sqrt{3}}\Big)\Big(e^{-1/\sqrt{3}}-e^{1/\sqrt{3}}\Big)$$
and
$$a=\Big(\frac{3}{2}-\frac{1}{2}e^{-1/\sqrt{3}}\Big)\Big(e^{1/\sqrt{3}}-e^{-1/\sqrt{3}}\Big)$$
Using these formulae you can calculate min.

5.
$$f''(x)=\delta'(x)-\delta(x)+3\delta(x-1)+h(x),$$
where $h(x)=0$ for $x<1$ and $h(x)=6x$ for $x>1$.