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**Svar till Tentamen TATA 27 Partial Differential Equations
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1.
$$u_x(0,t) = \frac{4\sqrt{kt}}{\sqrt{\pi}}$$

2.
$$u(x,y) = \cos \pi t \sin \pi x + \sum_{k=1}^{\infty} A_k \sin k\pi t \sin k\pi x$$

where
$$A_k = \frac{2((-1)^k - 1)}{k^2\pi^2}$$

3. Using the maximum principle for the function $v(x,t) = u - ((x-\alpha)^2 + (y-\alpha)^2)/4$, we obtain the inequalities $v(x,t) \le \max(-\alpha^2/4, -(1-\alpha)^2/4)$, which imply
$$u(1/2,1/2) \le (1/2-\alpha)^2/2 + \max(-\alpha^2/4, -(1-\alpha)^2/4).$$
Using this inequality for $\alpha = 1/2$ we arrive at the required result.

4. The Euler equation is
$$2u^{''} = u$$
and its general solution is
$$u = ae^{x/\sqrt{2}} + be^{-x/\sqrt{2}}.$$
Using the boundary conditions for u we obtain
$$u = e^{-x/\sqrt{2}}.$$
Therefore
$$\min = \int_0^{\infty} (\frac{1}{2} + \frac{x}{\sqrt{2}} + 1)e^{-\sqrt{2}x} dx = \sqrt{2}$$

5.
$$f^{''}(x) = -(e-2)\delta'(x-1) + (3-e)\delta(x-1) + h(x),$$
where $h(x) = -\sin x$ for $x < 0$, $h(x) = e^x$ for $0 < x < 1$ and $h(x) = 6x$ for $x > 1$.

6. Let u_1 and u_2 be two different solutions. We introduce the function $u = u_1 - u_2$. We have

$$\int_V (\Delta u) u dV = 0$$

Using Green's formula, we obtain

$$\int_V |\nabla u|^2 dV = \int_S u \frac{\partial u}{\partial \hat{n}} dV \, .$$

Since $\frac{\partial u}{\partial \hat{n}} = -\alpha u$ we have that

$$\int_V |\nabla u|^2 dV = -\alpha \int_S u^2 dV \, .$$

This implies $u = 0$ on S and $\nabla u = 0$ in V. Therefore $u = 0$.