Homework 6: The Calculus of Variations.

U denotes a bounded, open set of $\mathbb{R}^n$, with smooth boundary. All given functions are assumed smooth. Notations are defined as under Section 8.1.2.

Question. Find $L = L(p, z, x)$ so that the PDE

$$-\Delta u + D\phi \cdot Du = f \text{ in U}$$

is the Euler-Lagrange equation corresponding to the functional $I[w] := \int_U L(Dw, w, x)dx$. (Hint: Look for a Lagrangian with an exponential term.)