Homework 5 - Hamilton-Jacobi Equations

(a)

Show that $u(x) \coloneqq 1 - |x|$ is a viscosity solution of

$$\begin{cases} |u'| & = 1 \text{ in } (-1,1) \\ u(-1) & = u(1) = 0. \end{cases} \tag{1}$$

This means that for each $v \in C^\infty(-1,1)$, if $u-v$ has a maximum (minimum) at a point $x_0 \in (-1,1)$, then $|v'(x_0)| \leq 1$ (≥ 1).

(b)

Show that $\tilde{u} \coloneqq |x| - 1$ is *not* a viscosity solution of (1).

(c)

Show that $\tilde{u}$ is a viscosity solution of

$$\begin{cases} -|\tilde{u}'| & = -1 \text{ in } (-1,1) \\ \tilde{u}(-1) & = \tilde{u}(1) = 0. \end{cases} \tag{2}$$

(Hint: What is the meaning of a viscosity solution of (2)?)