

Graduate Course in Partial Differential Equations (MAI0133)
Spring Semester 2017
Homework 1

The questions below are taken from Evans. You may wish to look there, as he sometimes includes hints.

1.1 Let U and V be open sets, with $V \subset\subset U$. Show that there exists a smooth function ζ such that $\zeta \equiv 1$ on V and $\zeta \equiv 0$ near ∂U .

1.2 Prove directly that if $u \in W^{1,p}(U)$, $1 < p < \infty$ and U is the open interval $(0, 1)$, then

$$|u(x) - u(y)| \leq |x - y|^{1-1/p} \left(\int_0^1 |u'(t)|^p dt \right)^{1/p}$$

for almost every $x, y \in [0, 1]$.

Please feel free to look through Chapter 5 in Evans for supporting theory and further reading. I am happy to also discuss your thoughts/problems related to the theory or any other questions you decide to think about related to the course.