

Mathematical Methods for Social Scientists
Math 196 (Sec 45), Winter 2006
Revision Sheet for Mid-term 1

This mid-term will cover those sections in the text book which we have studied in class. The following questions are of the style you can expect in the exam.

- (1) Consider the linear system of two equations in three variables:

$$\begin{aligned}k_{11}X + k_{12}Y + k_{13}Z &= c_1; \\k_{21}X + k_{22}Y + k_{23}Z &= c_2.\end{aligned}$$

- (a) Define the solution set for the system above.
(b) Find the solution set for the system

$$\begin{aligned}X + Y - Z &= 6; \\X + 2Y - 3Z &= 2.\end{aligned}$$

- (c) Describe this set geometrically. Say what it means for a system to be inconsistent.

- (2) Find the solution set for the linear system

$$\begin{aligned}k_{11}X + k_{12}Y &= c_1; \\k_{21}X + k_{22}Y &= c_2.\end{aligned}$$

when the coefficients are given by the following.

- (a) $k_{11} = 1, k_{12} = 2, k_{21} = -1, k_{22} = 1, c_1 = 2$ and $c_2 = 4$
(b) $k_{11} = 1, k_{12} = 2, k_{21} = -3, k_{22} = -6, k_{11} = 1, c_1 = 2$ and $c_2 = 4$
(3) (a) Define the augmented matrix for a system of m linear equations in n variables.
(b) Define the three elementary operations we perform on the augmented matrix.
(c) Define what it means for a system to be in echelon form.
(4) Put the following augmented matrices into row-echelon form using Gaussian elimination.

- (a)

$$\left(\begin{array}{cc|c} 2 & 3 & 3 \\ -1 & 4 & -4 \end{array} \right)$$

- (b)

$$\left(\begin{array}{ccc|c} -2 & 1 & 1 & 1 \\ 6 & 2 & 5 & 0 \end{array} \right)$$

- (c)

$$\left(\begin{array}{cc|c} 2 & 3 & 5 \\ -2 & -3 & 1 \end{array} \right)$$

- (d)

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 0 & 1 & 16 \end{array} \right)$$

- (5) (a) Define the product AB of an $n \times m$ matrix $A = (a_{ij})_{ij}$ with an $m \times k$ matrix $B = (b_{ij})_{ij}$.
(b) What are the dimensions of the matrix AB defined above?

(c) Find a matrix M such that

$$\begin{pmatrix} 1 & 2 \\ 0 & 5 \end{pmatrix} M = \begin{pmatrix} 4 \\ 25 \end{pmatrix}.$$

What are the dimensions of M ?

(6) Find M such that $AM = B$ where A and B are given by the following.

(a)

$$A = \begin{pmatrix} 4 & 2 \\ 5 & 5 \end{pmatrix} \quad B = \begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}$$

(b)

$$A = \begin{pmatrix} 1 & 2 \\ 3 & -3 \end{pmatrix} \quad B = \begin{pmatrix} 6 & -1 \\ 1 & 2 \end{pmatrix}$$

(c)

$$A = \begin{pmatrix} -2 & 4 \\ 8 & 4 \end{pmatrix} \quad B = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$$

(d)

$$A = \begin{pmatrix} -9 & 3 \\ 6 & -3 \end{pmatrix} \quad B = \begin{pmatrix} 4 & 2 \\ 1 & 1 \end{pmatrix}$$