

**Mathematical Methods for Social Scientists**  
**Math 196 (Sec 49), Spring 2006**  
Revision Sheet for Mid-term 2

This mid-term will cover those sections in the text book which we have studied in class upto 5.3 and will concentrate on the material covered since the first mid-term. The following questions are of the style you can expect in the exam.

- (1) (a) Define what it means for a matrix to be upper triangular, lower triangular or diagonal.  
(b) If  $A$  and  $B$  are upper triangular matrices, what can you say about  $AB$  and  $A^t$ ?  
(c) Write

$$\begin{pmatrix} 2 & 3 \\ -2 & 4 \end{pmatrix}$$

as a product of elementary matrices.

- (2) (a) Give three conditions that are equivalent to a matrix  $A$  being invertible.  
(b) Give a formula for the inverse of a matrix  $A$ .  
(c) Calculate the inverse of

$$\begin{pmatrix} 2 & 6 & 3 \\ -2 & 8 & 4 \\ 0 & -2 & 0 \end{pmatrix}$$

using this formula.

- (d) Compute the determinant of

$$\begin{pmatrix} 2 & 3 & 4 & 1 \\ 1 & 3 & 2 & 3 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 4 & 9 \end{pmatrix}$$

- (3) (a) Define a subspace. Show that the set of solutions to a system of homogeneous equations is a subspace.  
(b) Define the row space and column space of a matrix. Explain why the row spaces of two row equivalent matrices are the same.  
(c) Define what it means for a set of vectors to be linearly independent.  
(d) Show that  $\{(0, 1, 1), (1, 0, 1), (1, 1, 0)\}$  is linearly independent.
- (4) (a) Define what a basis is for a vector space. Define the dimension of a vector space — what fact means this definition is sensible?  
(b) Give two conditions that are equivalent to being a basis.  
(c) Is the set  $\{(-1, 2, 1), (2, 3, 1), (7, 7, 2)\}$  linearly independent? Justify your answer. Find a basis for the span of this set.

It will be useful to review your homework and make sure you understand it. I will also ask questions from the text book.