Mathematical Methods for Social Scientists
Math 196 (Sec 55), Winter 2007
Revision Sheet for Mid-term 1

This mid-term will cover those sections in the text book which we have studied in class. The following questions are of the style you can expect in the exam.

(1) Consider the linear system of two equations in three variables:

$$k_{11}X + k_{12}Y + k_{13}Z = c_1;$$
$$k_{21}X + k_{22}Y + k_{23}Z = c_2.$$

(a) Define the solution set for the system above.
(b) Find the solution set for the system

$$X + Y - Z = 6;$$
$$X + 2Y - 3Z = 2.$$

(c) Describe this set geometrically. Say what it means for a system to be inconsistent.

(2) Find the solution set for the linear system

$$k_{11}X + k_{12}Y = c_1;$$
$$k_{21}X + k_{22}Y = c_2.$$

when the coefficients are given by the following.
(a) $k_{11} = 1$, $k_{12} = 2$, $k_{21} = -1$, $k_{22} = 1$, $c_1 = 2$ and $c_2 = 4$
(b) $k_{11} = 1$, $k_{12} = 2$, $k_{21} = -3$, $k_{22} = -6$, $k_{11} = 1$, $c_1 = 2$ and $c_2 = 4$

(3) (a) Define the augmented matrix for a system of m linear equations in n variables.
(b) Define the three elementary operations we peform on the augmented matrix.
(c) Define what it means for a system to be in echelon form.

(4) Put the following augmented matrices into row-echelon form using Gaussian elimination.
(a)
$$\left(\begin{array}{cc|c} 2 & 3 & 3 \\ -1 & 4 & -4 \end{array}\right)$$
(b)
$$\left(\begin{array}{ccc|c} -2 & 1 & 1 & 1 \\ 6 & 2 & 5 & 0 \end{array}\right)$$
(c)
$$\left(\begin{array}{cc|c} 2 & 3 & 5 \\ -2 & -3 & 1 \end{array}\right)$$
(d)
$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 0 & 1 & 16 \end{array}\right)$$

(5) (a) Define the product AB of an $n \times m$ matrix $A = (a_{ij})_{ij}$ with an $m \times k$ matix $B = (b_{ij})_{ij}$.
(b) What are the dimensions of the matrix AB defined above?
(c) Find a matrix M such that

$$\begin{pmatrix} 1 & 2 \\ 0 & 5 \end{pmatrix} M = \begin{pmatrix} 4 \\ 25 \end{pmatrix}.$$

What are the dimensions of M?

(6) Find M such that $AM = B$ where A and B are given by the following.
(a)
$$A = \begin{pmatrix} 4 & 2 \\ 5 & 5 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}$$
(b)
$$A = \begin{pmatrix} 1 & 2 \\ 3 & -3 \end{pmatrix} \qquad B = \begin{pmatrix} 6 & -1 \\ 1 & 2 \end{pmatrix}$$

(c)

$$A = \begin{pmatrix} -2 & 4 \\ 8 & 4 \end{pmatrix} \quad B = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$$

(d)

$$A = \begin{pmatrix} -9 & 3 \\ 6 & -3 \end{pmatrix} \quad B = \begin{pmatrix} 4 & 2 \\ 1 & 1 \end{pmatrix}$$

(7) (a) Find, if possible the inverses of the following matrices using Gauss-Jordan elimination.

(i)

$$\begin{pmatrix} -2 & 4 \\ 8 & 4 \end{pmatrix}$$

(ii)

$$\begin{pmatrix} 1 & 6 \\ 8 & 14 \end{pmatrix}$$

(iii)

$$\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$$

(iv)

$$\begin{pmatrix} -2 & 4 \\ 8 & 4 \end{pmatrix}$$

(b) Prove that there can at most one inverse of a matrix.

(8) (a) Define the rank of a matrix. What result make this definition a sensible one?

(b) State a condition which characterises when $A\mathbf{x} = \mathbf{b}$ has a solution. When would the solution be unique?