Mathematical Methods for Social Scientists
Math 195 (Sec 45), Autumn 2005
Revision Sheet for the Final Examination

The final will cover all the material covered in this course, and will not place particular emphasis on any part of the syllabus. The following questions cover material not covered in the previous two review sheets. In order to be prepared for the final you should as a bare minimum be able to do all the questions on all three review sheets.

1. (a) Use the method of Lagrange multipliers to maximise the Cobb-Douglas production function $f(x, y) = Cx^\alpha y^{1-\alpha}$ subject to the constraint $g(x, y) = x + y = k$. (Here α, C and k are all given constants.)

   (b) This is a harder version of question 44 from 15.8. Use the method of Lagrange multipliers to find the maximum of
   $$f(x_1, x_2, y_1, y_2) = x_1y_1 + x_2y_2$$
   subject to the constraints
   $$g(x_1, x_2, y_1, y_2) = x_1^p + x_2^p = 1 \quad \text{and} \quad h(x_1, x_2, y_1, y_2) = y_1^q + y_2^q = 1,$$
   where $1/p + 1/q = 1$ and $p, q \in (1, \infty)$.

   (c) Use part (b) to prove
   $$a_1b_1 + a_2b_2 \le (a_1^p + a_2^p)^{\frac{1}{p}} (b_1^q + b_2^q)^{\frac{1}{q}} .$$
   This is called Hölder's inequality. The Cauchy-Schwarz inequality is the special $p = q = 2$. Hint: Set
   $$x_i = \frac{a_i}{(a_1^p + a_2^p)^{\frac{1}{p}}} \quad \text{and} \quad y_1 = \frac{b_i}{(b_1^q + b_2^q)^{\frac{1}{q}}}$$
   for $i = 1, 2$.

2. (a) State Fubini's theorem.

   (b) Use Fubini's theorem to evaluate
   $$\iint_D \frac{x}{1 + xy}\, dA(x, y),$$
   where $R = [0, 1] \times [0, 1]$.

   (c) Evaluate
   $$\iint_D xye^{x^2y}\, dA(x, y),$$
   where $R = [0, 1] \times [0, 2]$.

3. Express the following double integrals as iterated integrals and evaluate them.

(a)

$$\iint_D \frac{2y}{x^2+1}\, dA(x,y),$$

where $D = \{(x,y)\,|\,0 \le x \le 1, 0 \le y \le \sqrt{x}\}$.

(b)

$$\iint_D e^{\frac{x}{y}}\, dA(x,y),$$

where $D = \{(x,y)\,|\,0 \le y \le 2, y \le x \le y^3\}$.