

Elementary Functions and Calculus III
Math 133 (Sec 42), Spring 2005
Problem Set 5 (final question)

For $a \in \mathbf{R}$, consider the function $f_a : \mathbf{R} \rightarrow \mathbf{R}$ defined by

$$f_a(t) = \begin{cases} 1, & \text{if } t \geq a; \\ -1, & \text{if } t < a. \end{cases}$$

1. Show that the improper integral $\int_{-\infty}^{+\infty} f_a(x) dx$ diverges.
2. Show that $\lim_{b \rightarrow +\infty} \int_{-b}^b f_a(x) dx$ converges (i.e. exists and is finite). What is its value?
3. Write a short paragraph explaining why you think we do not choose to define the improper integral $\int_{-\infty}^{+\infty} f(x) dx$, for any function f , as in part 2.