Elementary Functions and Calculus III
Math 133 (Sec 42), Spring 2005
Revision Sheet for Mid-term 2

This mid-term will cover all the material we have studied so far this quarter, but will obviously concentrate on that studied after the first mid-term. You should be able to recall the main theorems, tests and definitions, such as L'Hôpitals rule in it variant forms, and the integral, ratio and comparison tests. The following questions will be of the style you can expect in the exam, but remember, this does not cover all the techniques we have learnt, so look back over your notes and last year's mid-term too.

1. (a) State L'Hôpital's rule for indeterminate forms of the type $0/0$.
   (b) Use L'Hôpital's rule to evaluate the following limits.
      i. $$\lim_{x\to 0} \frac{\sin x}{x}$$
      ii. $$\lim_{x\to 0} \frac{e^x - 1}{x}$$
      iii. $$\lim_{x\to 0} \frac{e^x - e^{-x}}{2\sin x}$$
2. (a) Consider a function $f : (a, b] \to \mathbf{R}$ such that $\lim_{x\searrow a} |f(x)| = +\infty$. Define the improper integral $\int_a^b f(x)\,dx$. When do we say such a thing converges or diverges?
   (b) Use the definition above to calculate the following $\int_0^1 \frac{1}{x^p}\,dx$ for $p < 1$.
   (c) Prove whether or not the following improper integral converges or diverges: $\int_0^1 \frac{1}{\sqrt{1-x}}\,dx$.
3. (a) For a sequence $\{a_n\}_n$ define what it means to say the sequence converges to some limit l. What notation is used to denote this number?
   (b) Prove by using any method you know to show whether or not the sequence $\{a_n\}_n$ converges when a_n is defined as follows for all $n \in \mathbf{N}$. (i) $a_n = (-4)^{n+1}$, (ii) $a_n = (9/10)^n$, (iii) $a_n = \sin(2\pi n)$.
   (c) Suppose we knew that $a_n \le U$ for some $U \in \mathbf{R}$ and for all $n \in \mathbf{N}$. What other fact would enable us to conclude that $\{a_n\}_n$ converged.
4. (a) Define what we mean by saying $\sum_{n=1}^{\infty} a_n$ converges. What does it mean for this series to diverge?
   (b) State the integral test for a series. State carefully the required conditions.
   (c) Use the integral test to prove that $\sum_{n=1}^{\infty} e^{-n}$ converges. Does $\sum_{n=1}^{\infty} n^{-p}$ converge or diverge for $p > 1$?
5. State the ratio test and use to prove the following either diverge or converge. (i) $\sum_{n=1}^{\infty} 4^n/n!$, (ii) $\sum_{n=1}^{\infty} n^3/(n^5 + 2)$.