Elementary Functions and Calculus III
Math 133 (Sec 42), Spring 2005
Revision Sheet for Mid-term 1

Because of the nature of the material we have covered so far, this exam will be more computational in nature than my previous tests. You should practice all the techniques we have learnt for solving differential equations and integrating and be able to apply them. There are not that many theorems to memorise, but you should know the integration by substitution theorem and the integration by parts theorem. The questions below will be of the style you can expect in the exam, but remember, this does not cover all the techniques we have learnt, so look back over your notes too.

There will be a review session on Wednesday evening where I will answer any questions you have. I will announce the time and location when I have booked a room. Last year's mid-term is also on the course webpage. I will write up solutions for the mid-term but not for the questions below.

1. Consider the following differential equation with an initial condition.

$$\begin{cases} (x^4+2x^2+1)y'(x)+(4x^3+4x)y(x)=e^x, & \text{for all } x\in\mathbf{R}; \\ y(0)=5 \end{cases}$$

   (a) Compute the integrating factor for this first-order linear differential equation.

   (b) Use the above or any technique you know to find an expression for y.

2. Use a substitution of your choice to find the following anti-derivatives.

   (a) $\int \frac{6x}{\sqrt{4-2x^2}}\,dx$

   (b) $\int \frac{14e^{2x}}{5+7e^{2x}}\,dx$

   (c) $\int x^3\sqrt{x^4-5}\,dx$

3. (a) State the integration by parts theorem.

   (b) Use integration by parts twice to find $\int_0^{\frac{\pi}{2}} x^2\cos x\,dx$.

   (c) Find an anti-derivative of ln and state the domain where your formula is valid.

4. Use the appropriate trigonometric identity to evaluate the following.

   (a) $\int_{-2}^{2}\sqrt{4-x^2}\,dx$

   (b) $\int_4^{12}\frac{\sqrt{x^2-9}}{x^3}\,dx$

5. Find all possible positive functions y which satisfy the relationship $y'(x)=9y(x)$ for all $x\in\mathbf{R}$.