

Name: _____

Elementary Functions and Calculus III
Math 133 (Sec 22), Spring 2004

Mid-term 1
21st April 2004

Instructions: The total time allowed for this examination is 50 minutes. The use of notes, textbooks or calculators is prohibited. Write your answer to each question in the space provided below it. Should you require more space write on the reverse of the paper, labeling your answers clearly. Write your name at the top of this sheet. The maximum number of points available for each question or part question is shown in parentheses next to the question. There are 4 questions. You should attempt as many of the questions as you can. Partial credit may be given for incomplete answers.

1. Consider the following differential equation with an initial condition.

$$\begin{cases} (x^2 + 9)y'(x) + 2xy(x) = x^2 + 9, & \text{for all } x \in \mathbf{R}; \\ y(0) = 0 \end{cases}$$

- (10 pts) (a) Compute the integrating factor for this first-order linear differential equation.

- (10 pts) (b) Use the above or any technique you know to find an expression for y .

2. Use a method of your choice to find the following anti-derivatives.

(7 pts) (a) $\int \sin^5 x \, dx$

(6 pts) (b) $\int \frac{14e^{2x}}{5+7e^{2x}} \, dx$

(7 pts) (c) $\int x^{89} \sqrt{x^{90} - 5} \, dx$

(6 pts) 3. (a) State the integration by parts theorem for definite integrals.

(7 pts) (b) Use integration by parts to find $\int_0^{\frac{\pi}{2}} x \cos x \, dx$.

(7 pts) (c) Use integration by parts twice to find $A_x(e^x \cos x)$.

4. Use the appropriate trigonometric identity to evaluate the following integrals.

(10 pts) (a) $\int_{-4}^4 \frac{1}{\sqrt{16-x^2}} dx$

(10 pts) (b) $\int_4^{12} \frac{x}{\sqrt{x^2+9}} dx$ (Recall: $\sin(\cos^{-1} x) = \sqrt{1-x^2}$)