Name: ______________________________

Elementary Functions and Calculus II
Math 132 (Sec 22), Autumn 2004
Mid-term 1
30th January 2004

Instructions: The total time allowed for this examination is 50 minutes. The use of notes, textbooks or calculators is prohibited. Write you answer to each question in the space provided below it. Should you require more space write on the reverse of the paper, labeling your answers clearly. Write your name at the top of this sheet. The maximum number of points available for each question or part question is shown in parentheses next to the question. There are 4 questions. You should attempt as many of the questions as you can. Partial credit may be given for incomplete answers.

1. The figure shows a unit circle and a line joining the origin to the point (x, y). The length of the arc from $(1, 0)$ to (x, y) is denoted by θ.

(x, y)

θ

$(1, 0)$

(6 pts) (a) On the figure, label the lengths which define $\cos\theta$ and $\sin\theta$. Also give the definition of $\tan\theta$ below.

(7 pts) (b) Use the identity $\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$ to derive an identity for $\cos(2t)$.

(7 pts) (c) Use another identity you know to write $\cos(2t)$ in terms of powers of $\cos t$ only. State clearly the identity you are using.

(5 pts) 2. (a) i. In lectures we proved that $\lim_{x\to 0}(\sin x)/x$ and $\lim_{x\to 0}(1-\cos x)/x$ exist. State (without proof) the values we computed them to be.

(5 pts) ii. Since all the functions involved in the limits above are continuous, why can we not just plug in the value $x = 0$ to calculate the limit? Be careful to state the exact reason.

(b) Calculate the following limits using the results in part (a) and other standard results you know. Label clearly points at which you use these facts.

(5 pts) i.

$$\lim_{x\to 0} \frac{\tan x}{x}$$

(5 pts) ii.

$$\lim_{x\to 0} \frac{\cos\theta - \cos(\theta + x)}{x}$$

3. Define $f : [0,4] \to \mathbf{R}$ by $f(x) = 4x - 3 - x^2$ for all $x \in [0,4]$.

(10 pts) (a) Find all the critical points of f.

(5 pts) (b) What conditions do f and its domain satisfy that justify the existence of a (global) maximum value and (global) minimum value of f on $[0, 4]$.

(5 pts) (c) Find the (global) minimum value of f on $[0, 4]$. How do you know this is the minimum value?

(10 pts) 4. (a) State the mean value theorem.

(10 pts) (b) Use the mean value theorem to show that if $f : [a, b] \to \mathbf{R}$ is differentiable on (a, b), continuous on (a, b) and $f'(x) = 0$ for all $x \in (a, b)$ then there exists a constant D such that $f(x) = D$ for all $x \in [a, b]$.