

Elementary Functions and Calculus I
Math 131 (Sec 42), Autumn 2004
Practice Mid-term 2

1. Recall the definitions of the following.
 - (a) That a function $f : \mathbf{R} \rightarrow \mathbf{R}$ has a limit l at a point x .
 - (b) That a function $f : \mathbf{R} \rightarrow \mathbf{R}$ be continuous at a point x .
 - (c) That a function $f : \mathbf{R} \rightarrow \mathbf{R}$ be differentiable at a point x .
2. Recall the following results (without proving them). State clearly any assumptions that are required for each result to hold.
 - (a) The intermediate value theorem.
 - (b) The product rule.
 - (c) The quotient rule.
 - (d) The chain rule.
 - (e) The power rule.
3. Apply the intermediate value theorem to show that $p(x) = x^7 + 10x^3 + 2$ has at least one root.
4. Use any of the definitions or theorems you have quoted above to find the derivative f' of f when defined by the following formulae. State clearly which results you are using, and prove any steps which are not direct applications of the theorems.
 - (a) $f(x) = 10x^{25} + 50x^2 - 4x + 20x^3$
 - (b) $f(x) = (x^3 + 40x)^{308}$
 - (c) $f(x) = \frac{x^9 + 50}{x^2 + 4}$
5. Given that $y : \mathbf{R} \rightarrow \mathbf{R}$ is differentiable and satisfies $x^2 y(x) = 1 + y^2(x)x$ find $y'(c)$ in terms of c and $y(c)$.
6. Let $g : \mathbf{R} \rightarrow \mathbf{R}$ be defined by $g(x) = x(x+1)$ if $x \leq 0$ and $g(x) = ax + b$ if $x > 0$ for some given $a, b \in \mathbf{R}$. Find $g'(y)$ for any $y \neq 0$. For what values of a and b is g continuous at zero? For what values of a and b is g differentiable at zero? What must $g'(0)$ be if it exists? Explain your answer.